DAV PUBLIC SCHOOLS, ZONE – II, ODISHA

QUESTION BANK FOR CLASS – XII

(2015 – 16)

MATHEMATICS

CHAPTER – 1

RELATIONS AND FUNCTIONS

SECTION – A (One mark questions)

1. If $f:R \to R$ be defined by $f(x) = (3-x^3)^{\frac{1}{3}}$, then find $fof(x)$

2. Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. State if f is one-one or onto.
3. If the binary operation * on the set Z of integers is defined by a* b = a+b-5, then write the Identity element for the operation in * in Z
4. State the reason for the relation R in the set { 1,2,3} given by R = {(1,2),(2,1)} not to be transitive.
5. Let $A = \{1,2,3\}, B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B. State if f is one-one or onto

6. If $f: R \to R$and $g: R \to R$defined by $(x) = 27x^3$and $g(x) = x^{\frac{1}{3}}$, find $gof(x)$
7.If f :R–{0}→R–{0} is defined as f(x)=3/x, find f^{-1}(x).
8 .If f(x)=$\frac{5x+3}{4x-5}$, find f[f(x)].
9 .If n(A)=2 and n(B)=3. Find the number of function from A to B.
10. Write the number of equivalence relations on {1,2,3} containing (1,2).

SECTION – B & C (Four & Six mark questions)

1. Let A=N×N and * be the binary operation on A defined by (a,b)*(c,d)=(a+c, b+d) Show that * is commutative and associative. Find the identity element for * on A, if any.

2. Show that the function f in A=R-$\left\{\frac{2}{3}\right\}$ defined as f(x) =$\frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}(x)

3. Show that the relation R in the set A={x:X€z,0≤x≤12} given by={(a,b):|a-b| is divisible by 4} is an equivalence relation find the set of all the lements elated to1.

4. let * be a binary operations on Q defined by a*b=a+b - ab; a,b ∊ Q show that

i) Associative and ii) commutative

5. If R_1 and R_2 be two equivalence relations on a set A, prove that $R_1 \cap R_2$ I s also equivalence relation on A. What shall you tell about the equivalence of $R_1 \cup R_2$ (justify).

6. Let $f: N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$.Show that $f: N \to S$ is Invertible, where S is the range of f . Hence find inverse of f .

7. Let A = {1,2,3,…9} and R be a relation in A×A defined by (a,b) R (c,d) if a+d = b+c for all (a,b) , (c,d) in A×A . Prove that R is an equivalence relation .

8. Show that the function $f: R \to (-1,1)$ defined as $f(x) = \frac{x}{1+|x|}$ is one – one and onto function. Do you think it is invertible? If so then find f^{-1}

9. Define a binary operation * on A={0,1,2,3,4,5,} as

a*b=$\begin{cases} a+b \ \ if \ a+b<6 \\ a+b-6 \ , \ if \ a+b \geq 6 \end{cases}$ show that 0 is the identity for this operation and each elements ‘a’ of this set is invertible with 6-a being the inverse of ‘a’.

10. Show that f : N→N given by f(x)=$\begin{cases} x+1, if \ x \ is \ odd \\ x-1 \ , if \ x \ is \ even \end{cases}$, is bijective

(both one-one and onto).

CHAPTER – 2

INVERSE TRIGNOMATRIC FUNCTIONS

SECTION – A (One mark questions)

1. Write the principal value of $\sin^{-1}(\cos(\sin^{-1} ½))$.

2. What is the principal value of $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{6}\right) + \tan^{-1}\left(\tan\frac{7\pi}{6}\right)$?

3.Write the value of $\cot^{-1}(\frac{\pi}{4} - 2\cot^{-1} 3)$.

4.If$\sin(\sin^{-1}\frac{1}{5} + \cos^{-1} x) = 1$, then find the value of x .

5. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

6. Write the principal value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$.

7. If $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$ then what is the value of $\cot^{-1}x + \cot^{-1}y$.

8. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right) = \frac{\pi}{4} + \frac{1}{2}cos^{-1} x^2$

9. If $\sin(\sin^{-1}1/2 + \cos^{-1}x) = 1$,then find the value of x.

10.Write the range of one branch of $\sin^{-1}x$ other then the principal branch.

11.What is the principal value of $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{6}\right) + \tan^{-1}\left(\tan\frac{7\pi}{6}\right)$?

12. Prove the following: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}(1/3) = \frac{9}{4}\sin^{-1}(\frac{2\sqrt{2}}{3})$.

13. Solve for x : $\sin^{-1}6x + \sin^{-1}6\sqrt{3}x = -\pi/2$.

14. If $\cos^{-1}(x/2) + \cos^{-1}(x/3) = \alpha$,then prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$.

15. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

16. Solve the following equation: $cos(\tan^{-1} x) = sin\left(\cot^{-1}\frac{3}{4}\right)$

SECTION – B (Four mark questions)

1. Find the values of x which satisfy the equation $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$.

2. Solve for x , $2\tan^{-1}(sinx) = \tan^{-1}(2\sec x)$, $0<x<\frac{\pi}{2}$.

3.Solve the following equation $\tan^{-1}(\frac{x-1}{x-2}) + \tan^{-1}(\frac{x+1}{x+2}) = \frac{\pi}{4}$.

4.Prove that $\sin^{-1}(\frac{5}{13}) + \cos^{-1}(\frac{3}{5}) = 2\tan^{-1}(\frac{63}{16})$.

5. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$

6. Solve the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1} x\}] = 0$.

8. Prove that: $\sin[\cot^{-1}(\cos\{\tan^{-1} x\})] = \sqrt{\frac{x^2+1}{x^2+2}}$.

9. Prove that : $\tan^{-1}[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$

10. Solve for x: $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$.

11. If $\sin(\sin^{-1}1/2 + \cos^{-1}x) = 1$,then find the value of x.

12.Write the range of one branch of $\sin^{-1}x$ other then the principal branch.

13.What is the principal value of $\text{cosec}^{-1}\left(\text{cosec}\frac{\pi}{6}\right) + \tan^{-1}\left(\tan\frac{7\pi}{6}\right)$?

14. Prove the following: $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}(1/3) = \frac{9}{4}\sin^{-1}(\frac{2\sqrt{2}}{3})$.

15. Solve for x : $\sin^{-1}6x + \sin^{-1}6\sqrt{3}x = -\pi/2$.

16. If $\cos^{-1}(x/2) + \cos^{-1}(x/3) = \alpha$,then prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$

.17. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

18. Solve the following equation: $cos(\tan^{-1} x) = sin\left(\cot^{-1}\frac{3}{4}\right)$

CHAPTER – 3 & 4

MATRICES AND DETERMINANTS

SECTION – A (One mark questions)

1. If A is a square matrix of order 3 such that $|adj\ A| = 225$, find $|A^1|$.

2. What is the value of the following determinant ? $\Delta = \begin{vmatrix} 4 & a & b+c \\ 4 & b & c+a \\ 4 & c & a+b \end{vmatrix}$

3. If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, find the value of x and y.

4. If $\begin{bmatrix} y+2x & 5 \\ -x & 3 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ -2 & 3 \end{bmatrix}$, find the value of y.

5. For what value of x, the matrix [$\begin{matrix} 5-x & x+1 \\ 2 & 4 \end{matrix}$] is singular.

6. Construct 2X2 matrix A = [a_{ij}] whose elements is $a_{ij} = \frac{i}{j}$

7. If A is a square matrix such that $A^2 = A$, then write the value of $(I + A)^2 - 3A$.

8. If A and B are matrices of order 3 and |A| = 5,|B| = 3. Find the value of |3AB|.

9. . Write the value of x + y + z, If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

10. Evaluate $\begin{vmatrix} \text{Cos } 15 & \sin 15 \\ \sin 75 & \cos 75 \end{vmatrix}$

SECTION – B & C (Four & Six mark questions)

1. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

2. If A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

3. If A=$\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that A2 - 5A +7I=0, Hence find A^{-1}.

4. If A=$\begin{bmatrix} 0 & -\tan x/2 \\ \tan x/2 & 0 \end{bmatrix}$ and I is the identity matrix of order 2 show that

I+A=(I–A)$\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

5. If F(x)=$\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$,show that F(x)F(y)=F(x+y)

6. Using elementary transformations, find the inverse of the matrix $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$.

7. For the matrix A $=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$,show that $A^3 - 6A^2 + 5A + 11I = O. Hence, find A^{-1}$

8. Using properties determinants. Prove that

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

9. Using properties of determinates, prove that:

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix} = (a+b+c)(a-b)(c-b)$$

10. Prove without expanding that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$=abc+ ab+ bc+ ca

=abc$\left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{a}} + \frac{\mathbf{1}}{\boldsymbol{b}} + \frac{\mathbf{1}}{\boldsymbol{c}}\right)$ or$\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is a factor of determinant.

11. Using properties of determinants ,prove that

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

12. Using properties of determinants prove that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

13. If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$,a≠1 then prove by induction that A^n= $\begin{bmatrix} a^n & \frac{b(a^{n-1})}{a-1} \\ 0 & 1 \end{bmatrix}$

14. For wellbeing of orphanage, three trusts A, B and C was donated 10%, 15% and 20% of their total fund Rs. 200000, Rs.300000 and Rs. 500000 respectively. Using matrix multiplication finds the total amount of money received by orphanage by three trusts. By such donations, which values are generated?

15. A school wants to award its students for the value of honesty, regularity, and hard work with a total cash award of Rs. 6,000 .Three times the award money for hard work added to that given for honesty amounts to Rs 11,000. The award money given for honesty and hard work together is double the one f=given for regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values ,namely, honesty, regularity and hard work, suggest one more value which the school must include for awards.

16. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more values which the management of the colony must include for awards.

17. A trust caring for handicapped children gets Rs. 30000 every month from its donors. The trust spends half of the funds received for medical and educational care of the children and for that it charges 2% of the spent amount from them and deposits the balance amount in a private bank to get the money multiplied so that in future the trust goes on functioning regularly. What percent of interest should the trust gets from the bank to get a total of Rs. 1800 every month? Using matrix method, find the rate of interest. Do you think people should donate to such trusts?

18. There are 2 families A and B .There are 4 men ,6 women and 2 children in family A, and 2 men, ,2 women and 4 children in family B, The recommended daily amount of calories is 2400 for 1900 for women ,1800 for children and 45 gms of proteins for men ,55 grams for women and 33 grams for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of 2 families .What awareness can you create among people about the balanced diet from this question.

19. Two schools Aand B decided to award prizes to their students for three values honesty(x) , punctuality(y) and obedience (z) . School A decided to award a total of Rs. 11,000 for the three values to 5,4, and 3 students respectively , while school B decided to award Rs. 10, 700 for the three values to 4,3,and 5 students respectively .If all the three prizes together amount to Rs. 2700 , then using matrix method find the values of x,y, and z . Which value you prefer to be rewarded most and why .

20. Given A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, B= $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, Find BA and use this to solve the system of equations $y+2z=7$, $x-y=3$, $2x+3y+4z=17$.

21.Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ =3abc-a^2 –b^2 –c^2 .

22.Prove that $\begin{vmatrix} b2+c2 & a2 & a2 \\ b2 & c2+a2 & b2 \\ c2 & c2 & a2+b2 \end{vmatrix}$ =4a^2 b^2 c^2 .

23. Using properties of determinates,

Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & cb & c^2+1 \end{vmatrix}$ =(1+$a^2+b^2+c^2$)

24. Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$ =2(a +b+ c)^3 .

25. Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$ = 3abc – a^2 – b^2 – c^2 .

CHAPTER – 5

CONTINUTY AND DIFFERENTIABILITY

SECTION – A (One mark questions)

1. The function f(x) = $\begin{cases} \frac{sinx}{x} + cosx\ , & if\ x \neq 0 \\ k & ,if\ x = 0 \end{cases}$

Is continuous at x=0, then find the value of k.

2. If f(x) = sin x^0 , find $\frac{dy}{dx}$

3. If f(x) =|cos x|, find $f'\left(\frac{\pi}{4}\right)$

4. **Find the** derivative of $\log_{10}(\sin x)$

5. **Differentiate** $\mathbf{Sin^{-1}}\left(\frac{\mathbf{2^x}}{\mathbf{1+4^x}}\right)$ **w.r.t x**

SECTION – B&C (Four & Six mark questions)

1. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then show that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$.

2. If y = log $\left[x+\sqrt{1+x^2}\right]$, prove that $(1+x^2)\,d^2y/dx^2 + x\frac{dy}{dx} = 0$.

3. a) *If* $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2y'' + 2x(x^2+1)y' = 2$

b). If $F(x)=\frac{\sqrt{2}\cos x-1}{\cot x-1}, x\neq\frac{\pi}{4}$.find the values of $f\left(\frac{\pi}{4}\right)$ so that f(x) becomes continuous at $x=\frac{\pi}{4}$

4. Find a,b and c for which the function

$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx\sqrt{x}}, & x\geq 0\end{cases}$$ is continous at x=0

5. Find the value of k so that the function $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & if\ x\neq\pi/2\\ 3, & if\ x=\pi/2\end{cases}$ is continuous at x = $\pi/2$

6. The function $f(x)=\begin{cases}3ax+b, & if\ x>1\\ 11, & if\ x=1\\ 5ax-2b, & if\ x<1\end{cases}$ is continuous at x=1,

find the values of a and b.

7. Examine the following function f(x) for continuity at x = 1 and differentiability at x = 2, $f(x)=\begin{cases}5x-4, & 0<x<1\\ 4x^2-3x, & 1\leq x<2\\ 3x+4, & x\geq 2\end{cases}$

8. Verify mean value theorem for the function f(x) = (x-3)(x-6)(x-9) in [3,5].

9. Le $\boldsymbol{f(x)}=\begin{cases}\frac{\mathbf{1-\cos 4x}}{\boldsymbol{x^2}}, & \boldsymbol{if\ x<0}\\ \boldsymbol{a}, & \boldsymbol{if\ x=0}\\ \frac{\sqrt{\boldsymbol{x}}}{\sqrt{\mathbf{16}+\sqrt{\boldsymbol{x}}}-\mathbf{4}}, & \boldsymbol{if\ x>0}\end{cases}$ For what value of a,f is continuous at x = $\mathbf{0}$

10. Find the value of `a` for which the function f (x) defined as :

$$f(x) = \begin{cases} a \sin\frac{\pi}{2}(x+1), & x \le 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$, is continuous at x = 0.

11. If cos y=xcos(a+b) with a≠ ±1 prove that $\frac{dy}{dx} = \frac{cos^2(a+y)}{sina}$

12. .If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

13. Differentiate, $\sin^{-1} 2x\sqrt{1-x^2}$ $w.r.t$ $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$.

14. If $x = \sin t$ and $y = \sin pt$, Prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

15. If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, $showthat \frac{dy}{dx} = -\frac{y}{x}$

16. If y = $(\tan^{-1} x)^2$ Show that: $(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2$.

17. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$, prove that :$\frac{dy}{dx} = \frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$.

18. If $y = A e^{mx} + B e^{nx}$, prove that $d^2y/dx^2 - (m+n)\, dy/dx + mny = 0$.

19. If log y = $\tan^{-1}$, show that $(1+x^2)y_2 + (2x+1)y_1 = 0$.

20. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, show that $dy/dx = -1/(1+x)^2$

21. If y = 3cos(log x) + 4sin(log x), show that $x^2 (d^2y/dx^2) + x(dy/dx) + y = 0$.

22. If x= a(θ- sinθ), y = a(1+cosθ), find d^2y/dx^2.

23. If sin y = x sin(a+y), prove that dy/dx = $\sin^2(a+y)/\sin a$.

24. If y = $(\tan^{-1} x)^2$, prove that $(x^2 + 1)^2 (d^2y/dx^2) + 2x(x^2 + 1)\, dy/dx = 2$.

25.If y = $\sin^{-1}x$, show that $(1-x^2)\, d^2y/dx^2 - x\, dy/dx = 0$.

26. If y = log $\left[\boldsymbol{x} + \sqrt{\boldsymbol{1}+\boldsymbol{x^2}}\right]$, prove that $(1+x^2)\, d^2y/dx^2 + x\boldsymbol{\frac{dy}{dx}} = 0$.

27.If y = $\boldsymbol{\frac{\sin^{-1} x}{\sqrt{1-x^2}}}$, then show that $(1-x^2)\, d^2y/dx^2 - 3x\boldsymbol{\frac{dy}{dx}} - y = 0$.

28. If $y^x = e^{y-x}$, prove that dy/dx = $(1+\log y)^2/\log y$.

29. If $x^p y^q = (x+y)^{p+q}$, prove that (i) dy/dx = y/x and (ii) $d^2 y/dx^2$.

30. If $y = \left(x + \sqrt{x^2+1}\right)^m$ then show that $\left(x^2+1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$.

CHAPTER – 6

APPLICATIONS OF DERIVATIVES

SECTION – B (Four mark questions)

1. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.
2. Using differential, find the approximate value of $\sqrt{25.2}$
3.A particle moves along the curve $6y=x^3+2$ find the points on the curve at which y co-ordinates is changing 8 times as fast as the x co-ordinates.
4.Prove that $y=\frac{4sin\theta}{(2+cos\theta\ )}-\theta$ is an increasing of θ function in$(0,\frac{\theta}{2})$.
5.Find the equations of the tangents to the curves $y=\sqrt{3x-2}$ which is parallel to the line 4x- 2y+5=0
6.Find the angle between the curves $y^2=x$ and $x^2=y$.

7. Prove that the curves x y = 4 and $x^2 + y^2 = 8$ touch each other.

8. The length x of a rectangle is decreasing at the rate of 5 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of (a) the perimeter, (b) the area of the rectangle.

9. Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand-cone increasing, when the height is 4 cm?

10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

11. Find the interval in which the function f given by f(x) = sin x + cos x, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

12. Find the intervals in which the following functions are increasing or Decreasing:

i.$f(x)=8 + 36x + 3x^2 – 2x^3$.

ii.$f(x)= – 2x^3 – 9x^2 – 12x + 1$.

iii.$f(x)=x^4 - 4x^3 + 4x^2 + 15$.

13. Find the intervals in which the function f given by f(x) = sin x +cos x ,

$0 \leq x \leq 2\pi$, is strictly increasing or decreasing .

14. Prove that y = $\frac{4\sin\theta}{2+cos\theta} - \theta$ is an increasing function of θ in [0, π/2].

15. Find a point on the curve y = $(x-2)^2$, at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

16. Find the equation of the tangent to the curve x = $\sin 3t$, y =$\cos 2t$ at t = $\frac{\pi}{4}$

17. Prove that the curve x = y^2 and xy = k cut at right angles if $8k^2 = 1$.

18. Using differential, find the approximate value of $\sqrt{49.5}$.

19.Using differential, find the approximate value of f(2.01), where

f(x) = $4x^3 + 5x^2 + 2$.

20. Find the intervals in which the function f(x) = $x^3 + \frac{1}{x^3}$ is

(i) increasing (ii) decreasing.

SECTION – C (Six mark questions)

1. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.
2. A wire of length 28 m is to be cut into two half pieces. One of the pieces is to be made into a circle and the other into a square. What should be the lengths of two pieces so that the combined area of the square and the circle is minimum?
3. Prove that the volume of the greatest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.
4. Show that the rectangle of maximum area that can inscribed in a circle of radius r is a square of side $\sqrt{2}\,r$.
5. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h is $\frac{h}{3}$.
6. Show that the semi-vertical angle of a right circular cone of given total surface area and maximum volume is $\sin^{-1}\frac{1}{3}$
7. Show that a right circular cylinder which is open at the top, and has a given surface area will have the greatest volume if its height is equal to the radius of its base.
8. If the length of three sides of a trapezium other than the base are equal to 10 cm each, then find the maximum area of the trapezium.

9. Show that the height of cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{3}$. Also find the maximum volume.

10. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, then show that the area of the triangle is maximum, when the angle between the given side and the hypotenuse is $\frac{\pi}{3}$.

11. . A given quantity of metal is cast into a half cylinder with a rectangular base and semi circular ends. Show that the total surface area is minimum, if the ratio of the length of the cylinder to the diameter of its semicircular ends is $\pi : (\pi + 2)$.

12. An open box, with a square base, is to be made out of a given quantity of metal sheet of area c^2. Show that the maximum volume of the box is $c^3/6\sqrt{3}$.

13. Find the area of greatest rectangle that can be inscribed in an ellipse $x^2/a^2 + y^2/b^2 = 1$.

14. A point on the hypotenuse of a right angled triangle is at distances *a* and *b* from the sides. Show that the length of the hypotenuse is at least$\left(a^{2/3} + b^{2/3}\right)^{3/2}$.

15. An Apache helicopter of enemy is flying along the curve given by $y= x^2 +7$ .A soldier placed at (3,7) ,wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

16. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{\mathbf{2R}}{\sqrt{\mathbf{3}}}$. Also find the maximum volume.

17. A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is p cm , show that the window will allow the maximum possible light only when the radius of the semicircle is $\frac{\boldsymbol{p}}{\boldsymbol{\pi+4}}$ cm .

CHAPTER – 7

INTEGRALS

SECTION – A (One mark questions)

1. Evalute $\int_0^8 |x-5|dx$
2. Evalute$\int \frac{dx}{x+xlogx}$
3. Evalute$\int_0^{1.5}[x]dx$
4. Find the anti derivative of $\sqrt{x}+\frac{1}{\sqrt{x}}$.
5. Evalute $\int \frac{1}{x+x\log x}$ dx.
6. Evalute $\int \sec x\,(\sec x+\tan\ x)\,dx$.
7. Evaluate $\int \boldsymbol{\sec x^0\ tanx^0}$ dx
8. Evaluate : $\int \boldsymbol{\frac{1}{x+xlogx}dx}$
9. Evaluate:$\int_{\mathbf{0}}^{\mathbf{1}}\boldsymbol{\frac{dx}{1+x^2}}$.
10. Write the value of$\int_{\mathbf{-1}}^{\mathbf{1}}\boldsymbol{x^{17}\ cos^4\ x\ dx}$
11. If $\int_0^1(3x^2+\ 2x+k)dx=0$ find the value of k.
12. If $\int_0^a 3x^2\ dx=8$ write the value of a.
13. Evaluate: $\int_{-\pi/2}^{\pi/2}(sin^5$ x + 1) dx
14. Evaluate: $\int_2^3 \frac{1}{x}\ dx$

SECTION – B&C (Four & Six mark questions)

1. Evaluate: $\int \frac{\boldsymbol{(x^4-x)^{\frac{1}{4}}}}{\boldsymbol{x^5}}$dx.
2. Evaluate:$\int \left[\boldsymbol{\frac{1}{logx}}-\boldsymbol{\frac{1}{(logx)^2}}\right]\boldsymbol{dx}$
3. Evaluate $\int \frac{\cos\ 2x-\cos\ 2a}{\cos\ x-\cos\ a}$ dx.
4. Evaluate $\int \frac{\sin\ x+\cos\ x}{9+16\sin\ 2x}$ dx.
5. Evaluate: $\int e2x\left(\frac{1-\sin\ 2x}{1-\cos\ 2x}\right)$.
6. Evaluate. $\int \frac{(3\sin\ x-2)\cos\ x}{5-cos\,2\ x-4\sin\ x}$ dx.
7. Evaluate: $\int\{\frac{1}{logx}-\ \frac{1}{(logx\ )2}\}dx$.
8. Evaluate: $\int \frac{\sin\ 4\ x}{\sin\ x}$ dx.

9. Evaluate : $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

10 .Evaluate: $\int_0^{\frac{\pi}{2}} \log sinx\ dx$.

11. Evaluate: $\int(\sqrt{tanx}+\sqrt{\cot x})dx$.

12. Evaluate: $\int_{-1}^{\frac{3}{2}} |\ x \sin \pi x\ |\ \ dx$

13. Evaluate: $\int \frac{(x^4-x)^{\frac{1}{4}}}{x^5}$dx.

14. Evaluate: $\int_0^{\pi} \frac{x \quad dx}{a^2 \ cos^2 x +\ b^2 \ sin^2 x}$

15. Evaluate: $\int \frac{(3 \sin\ \theta -2) \cos\ \theta}{5-cos^2\theta\ -4\ sin^2\theta}\ d\theta$.

16. Evaluate: $\int \frac{1-x^2}{x(1-2x)}dx$.

7. Evaluate: $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$.

18. Evaluate: $\int_0^{\frac{\pi}{2}} \log \sin x\ \ dx$

19. Evaluate: $\int_1^4 (x^2-x\ )$ dx , By using limit of sums.

20. Evaluate: $\int \sqrt{\frac{a+x}{a-x}}$ dx.

21. Prove that $\int_0^{\pi/4} \log(1+\tan x)\ dx = \frac{\pi}{8}\ log\ 2$

22. Evaluate: $\int_0^{\pi/4} \frac{\sin\ x+\cos\ x}{9+16 \sin\ 2x}$ dx

23. Evaluate by limit of sum: (i). $\int_0^1 (\ 3x^2+2x+1)dx$ (ii). $\int_1^4 (\ x^2-\ x)dx$

(iii). $\int_1^3 (\ 3x^2+\ 2x)dx$ (iv). $\int_1^3 (2x^2+\ 5x)dx$

24. Evaluate $\int(3x-2)\sqrt{x^2+x+1}\ \ dx$

25. Evaluate: $\int \frac{1-x^2}{x(1-2x)}\ dx$

CHAPTER – 8

APPLICATIONS OF INTEGRALS

SECTION – C (Six mark questions)

1. Find the area of the shaded region enclosed between the two circles :

   $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.

2. Using integration find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.
3. Using integration find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.
4. Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$.
5. Using integration find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.
6. Find the area of the region included between the parabola $y = 3x^2/4$ and the line $3x - 2y + 12 = 0$.
7. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$.
8. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A(2, 0), B(4, 5) and C(6, 3).
9. Find the area of the region using integration $\{(x,y); x^2+y^2 \le 2ax, y^2 \ge ax,; x \ge 0, y \ge 0,\}$

10. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and

    the straight line $\frac{x}{4}+\frac{y}{3}=1$

11. Find the area of the region bounded by the parabolas $y^2=4ax$ and $x^2=4ay$

12. Using integration find the area of the region
    $\{(x,y): |x+2| \le y \le \sqrt{20-x^2}\}$
13. Using integration find the area of the region bounded by the parabola

    $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$

14. Find the area of the region enclosed between the two circles:

    $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

15. Find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$ by using Integration.
16. Using integration find the area of the region included between the parabola $\boldsymbol{4y = 3x^2}$ and the line $\boldsymbol{3x - 2y + 12 = 0}$

17. Using the method of integration, find the area of the region bounded by lines 2x + y = 4 , 3x – 2 y = 6 and x – 3y + 5 = 0.

18. Find the area of the region $\{ (x, y) : \ y^2 \leq 4x \ , \ 4x^2 + 4\, y^2 \ \leq \ 9 \ \}$.

19. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} + 1$ and the straight line x/3+y/2 = 1.

20. Using method of integration find the area bounded by the curve |x|+|y|=1.

CHAPTER – 9

DIFFERENTIAL EQUATIONS

SECTION – A (One mark questions)

1. Write the integrating factor of the differential equations (i) $\frac{dy}{dx}(x\ logx) + y = 2logx$

   (ii) $(1+ y^2)\, dx - (\tan^{-1} y - x)\, dy = 0$.

2. How many arbitrary constants are there in the particular solution of a differential equation of order 3?

3. **Find the order and degree of the given differential equation:**

   1. $x\frac{dy}{dx} + \frac{2}{\frac{dy}{dx}} = y^2$.

   2. $\sqrt{1-x^2}\,dx + \sqrt{1-y^2}\,dy = 0$

   3 . $(d^2y/dx^2)^2 + \cos(dy/dx) = 0$.

   4 . $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 5\frac{d^2y}{dx^2}$

   5. $y\left(\frac{dy}{dx}\right)^3 + x^3\left(\frac{d^2y}{dx^2}\right)^2 - xy = \sin x$

   6. $[\frac{d^2x}{dx^2}]^3 + [\frac{dy}{dx}]^2 + \cos[\frac{dy}{dx}] + 1 = 0.$

   7. $[\frac{d^2y}{dx^2}]^2 - 2[\frac{d^2x}{dx^2}] - \left[\frac{dy}{dx}\right] + 1 = 0.$

   8. $\left(\frac{d^2y}{dx^2}\right)^3 - 5\left(\frac{dy}{dx}\right) + 6 = 0$

   9. $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

10. $\frac{d^4y}{dx^4}$ – sin [$\frac{d^3y}{dx^3}$] = 0.

SECTION – B&C (Four & Six mark questions)

1. Show the differential equation$(1+e^{2x})dy+(1+y^2)e^x dx=0$ given that y=1 when x=0
2. Show that the given differential equations are homogenous and solve $x^2dy+y(x+y)dx=0$ given that y(1)=1
3. Solve the following differtinal equation $(y-x)\frac{dy}{dx}= x+2y$
4. Solve differential equation (xdy-ydx) ysin$(\frac{y}{x}) = (\mathrm{xdy} + \mathrm{ydx})\ \mathrm{y}\cos(\frac{y}{x})$
5. Solve differential equation $\cos^2x \frac{dy}{dx} + y = \tan x$
6. Solve differential equation $\frac{dy}{dx}$ + ytanx=2x+x^2tanx

7. Form the differential equation representing the family of ellipses having foci on x- axis and centre at the origin.

8. Solve the differential equation $(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{(x^2-1)}$

9. Form the differential equation of the family of circles touching the X-axis at origin.

10. Solve the differential equation: y dx + x log ($\frac{y}{x}$) dy – 2x dy = 0.

11. Find the particular solution of the differential equation: $(y - \sin x)\,dx + \tan x\,dy = 0$ satisfying the condition $y = 0$ $when\ x = 0$.

12. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right)$ = 3x + 4y given that y = 0 when x = 0.

13. Show that the differential equation $\left[\boldsymbol{x\sin^2\left(\frac{y}{x}\right) - y}\right]\boldsymbol{dx + xdy = 0}$ is homogeneous. Find the particular solution, given that $\boldsymbol{y = \frac{\pi}{4}}$ when $\boldsymbol{x = 1}$

14. Find the particular solution of the differential equation $\boldsymbol{\frac{dy}{dx} + x\cot y = 2y + }$ $y^2\cot y, y \neq 0$ given that $x = 0$ when $y = \frac{\pi}{2}$.

15. Prove that $x^2-y^2= c(x^2+y^2)^2$ is the general solution of differential equation $(x^3-3xy^2)dx=(y^3-3x^2y)\,dy$,where c is a parameter.

<u>CHAPTER – 10</u>

VECTOR ALGEBRA

SECTION – A (One mark questions)

1. Find the angle between the vectors: $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} - \hat{k}$.

2. If $\vec{a}$ =($2\hat{\imath}$-$\hat{\jmath}$+$3\hat{k}$) and $\vec{b}$=($6\hat{\imath}$+$\lambda\hat{\jmath}$+$9\hat{k}$) $\vec{a}||\vec{b}$. Find the value of λ

3. Find the value of λ for which the vector $\vec{a}$ =($3\hat{\imath}$+$\hat{\jmath}$-$2\hat{k}$) and $\vec{b}$ =($\hat{\imath}$+$\lambda\hat{\jmath}$-$3\hat{k}$) are Perpendicular to each other

4. If $\vec{a}$ = 2 $\hat{\imath}$ – 3 $\hat{\jmath}$ + 4$\hat{k}$, $\vec{b}$= $\hat{\imath}$ + 2 $\hat{\jmath}$ – 3 $\hat{k}$ and $\vec{c}$ = 3 $\hat{\imath}$ + 4 $\hat{\jmath}$ – $\hat{k}$,Find $\vec{a}$.($\vec{b}$ x $\vec{c}$).

5. Find the projection of the vector $\hat{\imath}$-$\hat{\jmath}$ on the $\hat{\imath}$+$\hat{\jmath}$

6. Write the position vector of the mid-point of the vector joining the points P(2,3,4) and Q(4,1,–2).

7. Let$\vec{a}$ and $\vec{b}$ be two vector such that | $\vec{a}$ |=3 and | $\vec{b}$ |=$\frac{\sqrt{2}}{3}$ and $\vec{a}$ X $\vec{b}$ is a unit Vector. Then what is the angle between $\vec{a}$ and $\vec{b}$.

8. Find a vector of magnitude 11 in the direction opposite to that of PQ where P and Q are the points (1, 3, 2) and (–1, 0, 8), respectively.

9. The 2 vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a triangle ABC. Find the length of the median through A.

SECTION – B&C (Four & Six mark questions)

1. Find the value of λ so that the vectors i+j+k ,2i+3j –k and –i +λj +2k are coplanar.

2.If sum of two unit vector is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3.Let $\vec{a}$ = i+4j+2k $\vec{b}$ =3i–2j+7k and $\vec{c}$ =2i–j+7k.Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}$. $\vec{d}$ =15.

4. A girl walk s 4 km towards west ,then she walks 3 km in the direction 30^0 east of north and stop Determine the girls displacement from her initial point of departure.

5. If sum of two unit vector is a unit vector. Prove that the magnitude of their difference is $\sqrt{3}$.

6. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$ and $\vec{c} + \vec{a}$ are coplanar.

7. Find the value of λ so that the vectors i+j+k ,2i+3j –k and –i +λj +2k are coplanar.

8.Let $\vec{a}$ = i+4j+2k $\vec{b}$ =3i–2j+7k and $\vec{c}$ =2i–j+7k.Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}$. $\vec{d}$ =15.

9. A girl walk s 4 km towards west ,then she walks 3 km in the direction 30^0 east of north and stop Determine the girls displacement from her initial point of departure.

10. If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ .Find a vector $\vec{p}$ which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, and $\vec{p} \cdot \vec{c} = 18$.

11. Let $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}, \vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$.

12. Dot product of a vector with vectors $\hat{\imath} - \hat{\jmath} + \hat{k},\ 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ and $\hat{\imath} + \hat{\jmath} + \hat{k}$ are respectively 4, 0 and 2. Find the vector.

13. Show that the four points A, B ,C and D with position vectors,$4\hat{\imath} + 5\hat{\jmath} + \hat{k}$, $-(\hat{\jmath} + \hat{k})$, $3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $4(-\hat{\imath} + \hat{\jmath} + \hat{k})$ respectively, are coplanar.

14. If $\vec{\alpha} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 4\hat{k}$,then express $\vec{\beta}$ as the sum of two vectors such that one is parallel to $\vec{\alpha}$ and other is perpendicular to $\overrightarrow{\alpha}$.

15. If the vector –i+j–k bisects the angle between the vector $\vec{c}$ and the vector 3i +4j, then find the unit vector in the direction of $\vec{c}$.

16. If the points (–1, –1, 2), (2, *m*, 5) and (3,11, 6) are collinear, find the value of *m*.

17. Using vectors, prove that cos (A – B) = cosAcosB + sinAsinB.

18 .Prove that in any triangle ABC, cos A= $\frac{b^2+c^2-a^2}{2bc}$, where *a*, *b*, *c* are the magnitudes of the sides opposite to the vertices A, B, C, respectively.

19. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$ and $|\vec{c}| = 13$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\overrightarrow{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$.

<u>CHAPTER – 11</u>

THREE DIMENSIONAL GEOMETRY

SECTION – A (One mark questions)

1.Find the distance of the point (a,b,c) from Y-axis.

2.Find the distance between the two parallel planes:

$\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} - 2\hat{k}) = 6$ and $\vec{r} \cdot (6\hat{\imath} - \widehat{3}\hat{\jmath} - 6\hat{k}) = 27$.

3.If the direction ratios of a line are 1, 1, 2, find the direction cosines of the line.

3.If a line makes an angle of 30°, 60°, 90° with the positive direction of *x*, *y*, *z*-axes, respectively, then find its direction cosines.

4. Find the distance of the plane 2x – y + 2z + 1 =0 from the origin.

5. Find the direction cosines of the line passing through the following points: (-2, 4, -5), (1, 2, 3).

6. Write the vector equation of the following line: $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$.

7. Write the intercept cut off by the plane 2x + y – z = 5 on x-axis.

8. What are the direction cosines of a line, which makes equal angles with the coordinate axes?

9. Find the distance of the plane 3x – 4y + 12z = 3 from the origin.

10. Find the direction cosines of the line: $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

11. Find the angle between the line $\frac{x+1}{2} = \frac{y}{2} = \frac{z-3}{6}$ and the plane 10x+2y–11z =0.

SECTION – B&C (Four & Six mark questions)

1. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ intersect each other.

2. Find the equation of the plane passing through the intersection of the planes 3x–y+2z-4=0 and x+y+z-2=0 and the point (2,2,1).

4. Find the shortest distance between the lines:

$\vec{r} = (1 - \lambda)\hat{\imath} + (\lambda - 2)\hat{\jmath} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{\imath} + (2\mu - 1)\hat{\jmath} - (2\mu + 1)\hat{k}$.

5. Find the equation of the plane through the points (2, 1, 0),(3,–2,–2)and(3, 1, 7).

6. Find the equation of a plane which is at a distance $3\sqrt{3}$units from origin and the normal to which is equally inclined to coordinate axis.

7.Find the equation of a plane which bisects perpendicularly the line joining the points A (2, 3, 4) and B (4, 5, 8) at right angles.

8. Find the shortest distance between the lines $\vec{r}$ =4i–j +λ(i+2j–3k) and $\vec{r}$ =i–j+2k +μ(2i+4j–5k)

9. Find the distance of the point (–1, –5, –10) from the point of intersection of the line: $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$ and the plane $\vec{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$.

10. Find the coordinates of the point where the line through the points A (3,4,1) and B(5,1,6) crosses XY-planes.

11.Find the distance between the points P(6,5,9) and the plane determined by the points A(3,–1,2), B(**5,2,4**) and C(–1,–1,6).

12. Find the equation of the plane passing through the line of intersection of planes $\vec{r}.(\hat{\imath} + 3\hat{\jmath}) - 6 = 0$ and $\vec{r}.(3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0$, whose perpendicular distance from origin is unity.

13. Show that the lines $\frac{x+\_3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ *and* $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane containing the lines.

14. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P (1, 2, 3).

15. Find the length and the foot of the perpendicular from the point P (7, 14, 5) to the plane 2x + 4y – z = 2. Also find the image of point P in the plane.

CHAPTER – 12

LINEAR PROGRAMMING
SECTION - C (Six mark questions)

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of mixture contain at least 8 units of vitamins A and 10 units of vitamins C .Food I contains 2 units/kg of vitamin A and 1 units/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 unit/kg of vitamin C. It costs Rs 50 per kg to purchase Food I and Rs 70 per kg to purchase Food II .Formulate this problem as a linear programming problem to minimize the costs of such a mixture.

2. An aeroplane can carry a maximum of 200 passengers. Profit of Rs.1000 is made on each executive class ticket and a profit of Rs.600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, atleast four times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit of the airline . What is the maximum profit? Do you think, more passengers would prefer to travel by such an airline than by other?

3. A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has maximum profit? Formulate this problem as a LPP given that the objective is to maximize the profit.
4. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistorsand 150 capacitors and is required to produce two types of circuits A and B.Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires10 resistors, 20 transistorsand 30 capacitors. If the profit ontype A circuit isRs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP sothat the manufacturer can maximize his profit.

5. A firm has to transport 1200 packages using large vans which can carry 200packages each and small vans which can take 80 packages each. The cost for engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

6. A dietician has to develop a special diet using two foods P and Q . Each packet (contains 30 g.) of food P contains 12 units of calcium,4 units of iron,6 units of cholesterol and 6 units of vitamin A .Each packet of same quantity of food Q contains 3 units of calcium,20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

7. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftmens time in its making while a cricket bat takes 3 hours of machine time and 1 hours of craftmans time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftmans time. If the profit on a racket and on a bat is Ra 20 and Rs 10 respectively, find the number of tennis racket and cricket bats that the factory must manufacture to earn the maximum profit. Make it as LPP and solve it graphically.

8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25,000 and Rs 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70lakhs and his profit on the desktop model is Rs 4,500 and on the portable model is Rs 5,000. Make an LPP and solve it graphically.

9. A company produces softdrinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B go into each bottle of drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs 10. The supplier T has a packet of mix unit of A and 1 unit of B that costs Rs 4. How many packets of mixes from S and T should the company purchase to honour the contract requirements and yet minimise cost? Make an LPP and solve graphically.

10. A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. The control weeds, a liquid herbicide has to be used for crops A and B at the rates of 20 litres and 10 litres per hectare respectively. Further not more than 800 liters of herbicide should be used in order to protect fishes and wild life using a pond which collects drainage from this land. Keeping in mind that the protection of fish and wild life is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wild life is utmost necessary to preserve the balance in environment?

11. A dealer in rural area wishes to purchase a number of sewing machine. He had only **Rs**.5760 to invest and has space for at most 20 items. An electronic sewing machine cost him Rs.360 and a manually operated sewing machine Rs.240. He can sell an electronic sewing machine at a profit of Rs.22 and a manually operated sewing machine at a profit of Rs.18. Assuming that he can sell all the items that he can buy, how should he invest his money inorder to maximize his profit? Formulate this problem as LPP and solve graphically. Keeping rural background in mind justify the value up to the promoted for the selection of the manually operated machine.

12. One kind of flour cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of other ingredients used in making the cakes. Formulate the above as a linear programming and solve graphically.

CHAPTER – 13

PROBABILITY

SECTION – A(One mark questions)

1. 1.If P(A)=1/2 ,P(B)=0,then find P(A/B).
2. Write the probability of an even prime number on each die,when a pair

   of dice is rolled.
3. Two independent events A andB are given such that P(A)=0.3 and

   P(B)=0.6 find P(A and notB)

4. If X has a binomial distribution B(6,1/3) write P(x=1)

5. An urn contains 10 white and 3 black ball another urn contains 3 white and 5 balss and 2 ball are drawn from the 1st urn and put into the 2nd urn and then a ball is drawn from the 2nd urn. Find the probability that it is a white ball.

SECTION – B&C (Four & Six mark questions)

1. In a hurdle race Ritam has to cross 10 hurdles. The probability that he will clear all the hurdles is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles? Do you think that sports play an important role in student's life?

2. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and Standard deviation of the number of kings.

3. Out of 9 outstanding students of a school, there are 4 boys and 5 girls. A team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

4. In a game, a man earns a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find expected value of the amount he wins/loses.

5. In answering a questions on a MCQ test with 4 choice per question, a student's known the answer guesses or copies the answer. Let $\frac{1}{2}$ is the probability that he knows the answer, $\frac{1}{4}$ is the probability that he copies it..Assuming that the student who copies the answer will be correct has the probability, $\frac{3}{4}$ what is the probability that the student knows the answer given that he answered it correctly?

6. Assume that the chances of a patient having a hart attack is 40%, assuming that Meditation and yoga course reduces the risk of hart attack by 30% and prescription of certain drug reduces the risk by 25%.At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random, suffers hart attack. Find the probability that the patient followed a course of meditation and yoga.

7. The probabilities of A, B and C solving a problem are $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them can solve it.

8. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of die.

9. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up tails 25% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows head, what is the probability that it was from the two headed coin?

10. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

11. A manufacturer has three machine operators A,B and C . The first operator A produces 1% defective items, where as the other two operators B and C produce 5% and 7 % defective items respectively. A is in the job for 50% of the time, B is In the job for 30% of the time and C is in the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

12. Suppose that the reliability of a HIV test is specified as follows:
Of people having HIV, 90% of the test detect the diseases but 10% go undetected. Of people free of HIV, 99% of the tests are judge HIV –ive but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% has HIV , one person is selected at random, given the HIV tests and the pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV +ve ?

13. In a group of 400 people, 160 are smokers and non-vegetarian, 100 are smokers and vegetarian and the remaining are non-smokers and vegetarian. The probabilities of getting a special chest disease are 35%, 20% and 10% respectively. A person chosen from the group at random and is found to be suffering from the disease. What is the probability that the selected person is a smoker and non-vegetarian? What value is reflected in this question?

14. In a hurdle race, a player has to cross 10 hurdles .The probability that he will clear each hurdle is 5/6 .What is the probability that he will known down fewer than 2 hurdles?

15. 10% of the bulbs produced in a factory are of red color and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

****